Fall 2010

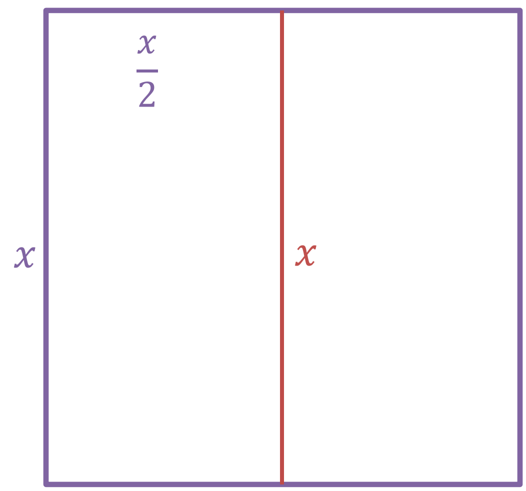
1.

$$Rectangle\ perimeter = 2x + 2\left(\frac{x}{2}\right) = 36$$

$$3x = 36$$

$$x = 12$$

$$Area = x^2 = 144$$



So the answer is **D. 144**

2.

Letting M be the original cost of the milk and B being the original cost of the bread.

$$M = 1.5B$$

$$\frac{M}{B} = 1.5$$

$$1.2M = 1.25xB$$

$$\frac{1.2}{1.25}\frac{M}{B} = x$$

$$\frac{1.2}{1.25}(1.5) = x$$

$$1.44 = x = 144\%$$

So the answer is **C. 144**

3.

$A = 9B$

$(90 - B) = 9(90 - A)$

$90 - B = 9(90 - 9B)$

$90 - B = 810 - 81B$

$80B = 720$

$B = 9$

So the answer is **C. 9°**

4.

$\frac{x-3}{x^2-10x-24}$ is undefined when $x^2 - 10x - 24 = 0$

$(x - 12)(x + 2) = 0$

$x = 12 \text{ } or \text{ } x = -2$

Multiply:

$12(-2) = -24$

So the answer is **B. -24**

5.

If any die is even, the product will be even, so we are looking for the probability that at least one die is even. This is the complement of all three dice are odd. Since the probability of any die being odd is $\frac{1}{2}$, we get:

$P(product\ is\ even) = 1 - \left(\frac{1}{2}\right) * \left(\frac{1}{2}\right) * \left(\frac{1}{2}\right) = 1 - \frac{1}{8} = \frac{7}{8}$

So the answer is **E.** $\mathbf{\frac{7}{8}}$

6.

$f(x) = ax^2 + bx + c$

$f(-1) = a(-1)^2 + b(-1) + c = 10$

$f(0) = a(0)^2 + b(0) + c = 5$

$f(1) = a(1)^2 + b(1) + c = 4$

$a - b + c = 10$

$c = 5$

$a + b + c = 4$

Subtracting the 3rd equation from the 1st;

$-2b = 6$

$b = -3$

Substitute into the 1st equation;

$a - (-3) + 5 = 10$

$a + 8 = 10$

$a = 2$

So $f(x) = 2x^2 - 3x + 5$

$f(2) = 2(2)^2 - 3(2) + 5 = 8 - 6 + 5 = 7$

So the answer is **A. 7**

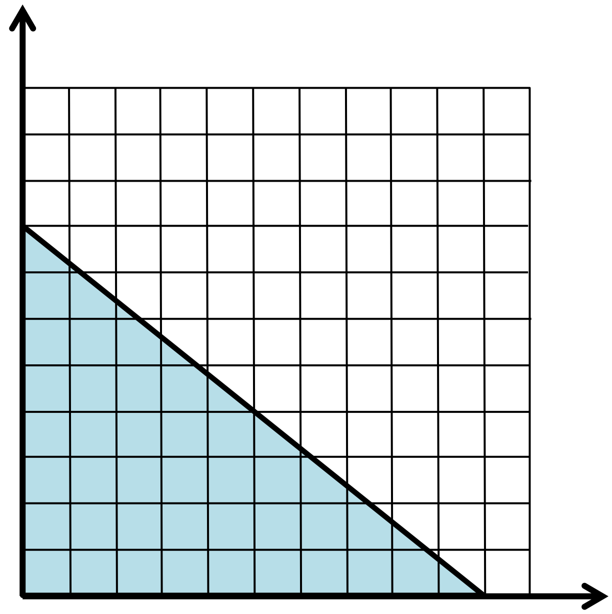
7.

Using (0,8) and (10,0) to find the slope:

$m = \frac{8-0}{0-10} = -\frac{4}{5}$

With the y intercept at 8, we can write the line as;

$y = -\frac{4}{5}x + 8$



Make a table:

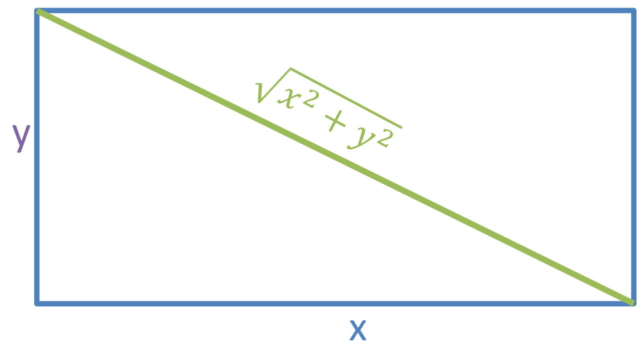
Notice that there is a lattice point at 0 for each x, so you must floor each y value to get the lattice points below the line and then add one because of the point at 0.

x	Y	# of lattice pts.
0	8	9
1	7.2	8
2	6.4	7
3	5.6	6
4	4.8	5
5	4	5
6	3.2	4
7	2.4	3
8	1.6	2
9	.8	1
10	0	1

Total: $9 + 8 + 7 + 6 + 5 + 5 + 4 + 3 + 2 + 1 + 1 = 51$

So the answer is **A. 51**

8.



Diagonal $= \sqrt{x^2 + y^2} = 20$

Perimeter $= 2x + 2y = 52$

$x + y = 26$

$(x + y)^2 = x^2 + 2xy + y^2 = 26^2 = 676$

From the diagonal: $x^2 + y^2 = 400$

$2xy = 276$

$xy = 138 =$ Area

So the answer is **B. 138**

9.

Start with the single digit #'s		Total
.2468	4 digits	4
Then the 2 digit #'s		
1012141618	10 digits	14
Repeated 8 times for 20-98	80 digits	94
Then the 3 digit #'s		
100102104106108		
*10 to get to 198	150 digits	244
Repeat that 8 times to get to 998	1200 digits	1444
Now the 4 digit #'s		
10001002100410061008		
*10 to get to 1098	200 digits	1644
Another 200 to get to 1198	200 digits	1844

Another 200 would put us over. Since the numbers end on multiples of 4 and 2010 is 2 away from a multiple of 4, It will be on a 2 in 12_ _ (specifically1**2**82).

So the answer is **A. 2**

10.

The lowest possible # that fits the AMATYC scheme is 101234

If we divide that by 35, we get 2892.4, so our number will be at least $35 * 2893$.

If you have a TI-83, you can simplify this process by pressing Y= and putting in 35x, then going to tblset and starting at 2893, then looking at the table for a value that fits the AMATYC scheme, otherwise you can work it out manually.

$2893 * 35 = 101255$	Y=C, doesn't work
$2894 * 35 = 101290$	M=C, doesn't work
$2895 * 35 = 101325$	works

So 101325 is the first multiple of 35 that fits

The last 2 digits are 25.

So the answer is **A. 25**

11.

$f(x) = \ln\left(x + \sqrt{1 + x^2}\right)$

$f(x) = \ln(7)$

$\ln 7 = \ln\left(x + \sqrt{1 + x^2}\right)$

$7 = x + \sqrt{1 + x^2}$

$7 - x = \sqrt{1 + x^2}$

$49 - 14x + x^2 = 1 + x^2$

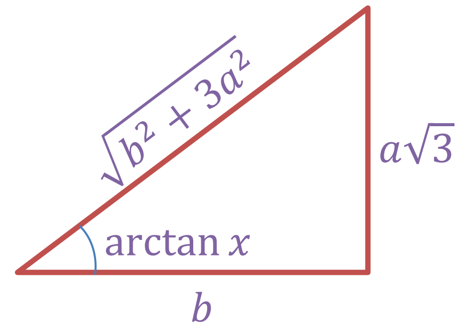
$49 - 14x = 1$

$-14x = -48$

$x = \frac{48}{14} = \frac{24}{7}$

So the answer is **B.** $\mathbf{\frac{24}{7}}$

12.



$$\sin(30° + \arctan x) = \frac{13}{14}$$

$$x = \frac{a\sqrt{3}}{b}$$

Use the property of sin of a sum.

$$\sin(30° + \arctan x) = (\sin 30°)(\cos(\arctan x)) + (\cos 30°)(\sin(\arctan x)) = \frac{13}{14}$$

$$\frac{13}{14} = \frac{1}{2}\cos(\arctan x) + \frac{\sqrt{3}}{2}\sin(\arctan x)$$

Using the triangle pictured:

$$\frac{13}{14} = \frac{1}{2}\frac{b}{\sqrt{b^2+3a^2}} + \frac{\sqrt{3}}{2}\frac{a\sqrt{3}}{\sqrt{b^2+3a^2}}$$

$$\frac{13}{14} = \frac{b+3a}{2\sqrt{b^2+3a^2}}$$

$$\frac{13}{7} = \frac{b+3a}{\sqrt{b^2+3a^2}}$$

$$13\sqrt{b^2+3a^2} = 7b + 21a$$

Square both sides: $169(b^2 + 3a^2) = 49b^2 + 294ab + 441a^2$

Divide by b^2: $169 + 507\left(\frac{a}{b}\right)^2 = 49 + 294\left(\frac{a}{b}\right) + 441\left(\frac{a}{b}\right)^2$

$$66\left(\frac{a}{b}\right)^2 - 294\left(\frac{a}{b}\right) + 120 = 0$$

$$11\left(\frac{a}{b}\right)^2 - 49\left(\frac{a}{b}\right) + 20 = 0$$

$$\frac{a}{b} = \frac{49 \pm \sqrt{49^2 - 4(11)(20)}}{2(11)} = \frac{49 \pm \sqrt{1521}}{22} = \frac{49 \pm 39}{22}$$

Since $\frac{a\sqrt{3}}{b} < 1, \frac{a}{b} = \frac{49-39}{22} = \frac{10}{22} = \frac{5}{11}$

$$5 + 11 = 16$$

So the answer is **A. 16**

13.

$$a^2 + b^2 + c^2 = 2010$$

$$b^2 = 2010 - a^2 - c^2$$

$$b = \sqrt{2010 - a^2 - c^2}$$

If you have a TI-83 or better, you can put this formula in with c as the variable, changing a manually. Press Y=, then put in $Y_1 = \sqrt{2010 - 1^2 - X^2}$, and $Y_2 = \sqrt{2010 - 2^2 - X^2}$. Now check the table:

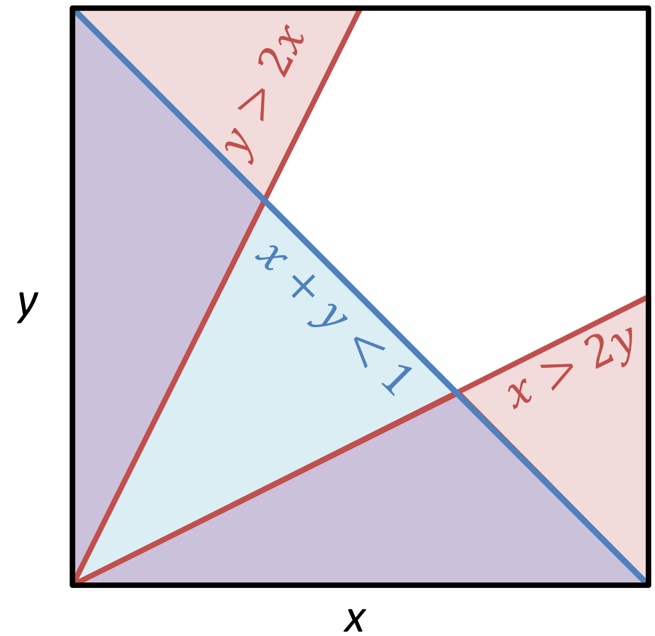
Notice that when a is 1, and c is 28, b is 35.

28 and 35 have a common factor of 7.

So the answer is **D. 7**

X	$Y_1 = \sqrt{2010 - 1^2 - X^2}$	$Y_2 = \sqrt{2010 - 2^2 - X^2}$
1	44.81071	44.77723
2	44.77723	44.74371
3	44.72136	44.68781
4	44.64303	44.60942
5	44.54211	44.50843
6	44.41846	44.38468
7	44.27189	44.23799
8	44.10215	44.06813
9	43.909	43.87482
10	43.6921	43.65776
11	43.45112	43.41659
12	43.18565	43.1509
13	42.89522	42.86024
14	42.57934	42.54409
15	42.23742	42.2019
16	41.86884	41.833
17	41.47288	41.4367
18	41.04875	41.01219
19	40.59557	40.5586
20	40.11234	40.07493
21	39.59798	39.56008
22	39.05125	39.01282
23	38.47077	38.43176
24	37.85499	37.81534
25	37.20215	37.16181
26	36.51027	36.46917
27	35.77709	35.73514
28	35	34.95712
29	34.17601	34.1321
30	33.30165	33.25658

14.



This problem is most easily done if you look at the 2 random numbers as a point on a 1 X 1 square. Then the conditions become

$x + y < 1$

AND

$(x > 2y \text{ OR } y > 2x)$,

which correspond to the purple triangles in the picture.

The bottom triangle has a base of 1, and to find the height, see where the 2 lines meet.

$x = 2y$

$x + y = 1$

$2y + y = 1$

$3y = 1$

$y = \frac{1}{3}$

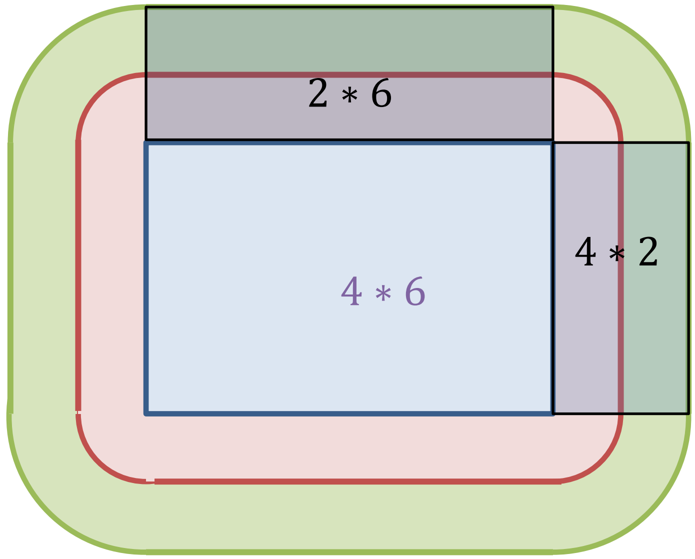
So the area of the triangle is $\frac{1}{2} * \frac{1}{3} * 1 = \frac{1}{6}$.

The other triangle is similar and also has area of $\frac{1}{6}$.

Together, they have an area of $\frac{1}{3}$, and the total area of the square is 1. So $\frac{1}{3}$ of the points in the square meet the conditions.

So the answer is **C.** $\mathbf{\frac{1}{3}}$

15.



The shape created will be a rounded rectangle as shown. We can split this into the center rectangle, with area of 24, the 2 side rectangles with area of 8 each, the top and bottom rectangles with area 12 each, and the corners can join into a circle of radius 2, for an area of 4π.

$$24 + 2 * 8 + 2 * 12 + 4\pi \approx 76.56 \approx 77$$

So the answer is **D. 77**

16.

$$f(x) = \frac{\sqrt{x^2-1}}{x}$$

$$f\big(f(x)\big) = f\left(\frac{\sqrt{x^2-1}}{x}\right) = \frac{\sqrt{\left(\frac{\sqrt{x^2-1}}{x}\right)^2 - 1}}{\frac{\sqrt{x^2-1}}{x}} = \frac{\sqrt{\frac{x^2-1}{x^2}-1}}{\frac{\sqrt{x^2-1}}{x}} = \frac{\sqrt{1-\frac{1}{x^2}-1}}{\frac{\sqrt{x^2-1}}{x}} = \frac{\sqrt{-\frac{1}{x^2}}}{\frac{\sqrt{x^2-1}}{x}}$$

Since $-\frac{1}{x^2}$ is always negative, the function always gives imaginary values, and never gives real values.

So the answer is **E. No values of x**

17.

Arithmetic sequence: Geometric sequence:

$a_1 = A,$ $b_1 = B$

$a_2 = A + r,$ $b_2 = Br$

$a_3 = A + 2r$ $b_3 = Br^2$

$A + B = 7$

$A + r + Br = 26$

$A + 2r + Br^2 = 90$

$-2A - 2r - 2Br = -52$

$-A + Br^2 - 2Br = 38$

$A = 7 - B$

$-A = B - 7$

$B - 7 + Br^2 - 2Br = 38$

$Br^2 - 2Br + B = 45$

$B(r-1)^2 = 3^2 * 5$

Since B and r are integers, B must be 5 and $r - 1$ must be 3

$r - 1 = 3$

$r = 4$

So the answer is **B. 4**

18.

Since the first and second terms are prime, and the others are just multiples of them, the first and second numbers have to be the prime factors of 12,500,000, which are 2 and 5.

There are 2 possibilities, either 2 is 1st and 5 is 2nd or 5 is 1st and 2 is 2nd. Let's quickly work through each to 7 terms:

$a_1 = 2$	$b_1 = 5$
$a_2 = 5$	$b_2 = 2$
$a_3 = 10$	$b_3 = 10$
$a_4 = 50$	$b_4 = 20$
$a_5 = 500$	$b_5 = 200$
$a_6 = 25000$	$b_6 = 4000$
$a_7 = 125{,}000{,}000$	$b_7 = 800{,}000$

Since the 8th term is the 6th term times the 7th term, the 8th term divided by the 7th term is the 6th term, which is 25000

So the answer is **E. 25,000**

19.

$P(2) = 77$

$P(P(2)) = P(77) = 1{,}874{,}027$

Since all of the coefficients are positive, we cannot have any powers of x leading to values higher than the function values.

$77^4 = 35{,}153{,}041$, so there cannot be any x^4 or higher powers.

$77^3 = 456{,}533$, so there are at most $4x^3$.

I there are $3x^3$, then that would leave more than $77(77)^2$, or $77x^2$, to make up the difference. Since that would put $P(2)$ over 77, there must be $4x^3$.

That leaves $77 - 4(2)^3 = 45$ for $P(2)$ and $1{,}874{,}027 - 4(77)^3 = 47{,}895$

$77^2 = 5929$, which goes into 47,895 8 times, so there is at most $8x^2$. By a similar argument to before, there must be exactly $8x^2$.

This leaves $47895 - 8(77)^2 = 463$

77 goes into 463 6 times, so there is 6x.

$463 - 6(77) = 1$, so 1 is the constant.

So $P(x) = 4x^3 + 8x^2 + 6x + 1$

To verify for 2, $P(2) = 4(2)^3 + 8(2)^2 + 6(2) + 1 = 32 + 32 + 12 + 1 = 77$

$4 + 8 + 6 + 1 = 19$

So the answer is **E. 19**

20.

What the question can be translated as "What is the minimum value of $|x-1|+|x-2|+\cdots|x-2010|$"

In order to do this, we want the average distance from x to {1,2,3,…2010} to be as low as possible, so it would make sense for x to be in the middle of those values, so let's try 1005.

$|1005-1|+\cdots|1005-1004|+|1005-1005|+|1005-1006|+\cdots|1005-2010|$

$1004+1003+\cdots 1+0+1+2+\cdots 1004+1005$

Using the formula for arithmetic sequences: $\frac{1004(1005)}{2}+\frac{1005(1006)}{2}=\frac{2010(1005)}{2}=1005^2$

To verify that this is the lowest value, choosing 1004 for x moves the sequence to $1003+1002+\cdots 1+0+1+\cdots 1005+1006=$ $\frac{1003(1004)}{2}+\frac{1006(1007)}{2}$, which is larger than 1005^2

If we use 1006, we get 1005^2, and if we use any other number, we get something larger.

So the answer is **B. $\mathbf{1005^2}$**